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PROBABILITIES OF TRAFFIC LOADS ON BRIDGES

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PROBABILITIES OF TRAFFIC LOADS ON BRIDGES

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SYNOPSIS

This paper considers for any determinate load length what design intensity of lane load can be logically inferred from the largest design vehicle and from the average long lane load. Design loads are determined so that the probabilities for exceeding them are made extremely small but not zero.

Broad Phases of the Bridge Loading Problem

There is a natural feeling that traffic loads per unit length of lane are smaller on long bridge spans than on short. Observations on actual traffic lanes corroborate this feeling but no rational analysis of the proper magnitude of the reduction seems to exist. Most bridge specifications reduce design traffic load intensities for long lengths of loading: then reduction is largest where economic considerations play an important role, and is less in countries whose officials are essentially guided by what other countries do.

A proper choice of design traffic load for highway bridges is economically important. If the design load is small, heavy traffic must be turned away. If it is too large, improductive construction costs will be required. Both alternatives imply economic losses. These can amount to millions of dollars on single projects.

The criterion of the general loading problem would at first sight seem to be the "safety factor" or "overload capacity" of the bridge. In brief, the "safety factor" is rationally regarded as a divisor of the nominal "yield strength" (or ultimate strength) resulting in a "working stress", which is so low that under that stress the probability of failure of the structure due to the scattering of the yield strength, uncertainties in the design calculations etc. is reduced below an extremely small but finite value, say 10^{-8}.

The probability of destruction can be made very small by reducing the working stress, thereby increasing the costs of construction. An economic probability of failure exists where the initial construction cost, the reconstruction cost due to this probability, and the capitalized maintenance is a minimum. A material failure in a secondary member, say a stringer, may imply local repairs only, and in some primary members total destruction and loss of life. The economic probability of failure may thus turn out higher for some members in a structure than for others.

The author has not seen such calculations carried through, but their basic ideas obviously form the essential point in any judicious choice of "safety" factors and working stresses. It is essential to bear in mind that the latter notions fundamentally do not refer to safety but to unsafety.

A "safety" factor concept, thus interpreted, can properly form the basis for determining the design traffic load for short spans which can be fully loaded

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by one or two sporadic excessively heavy vehicles. For long spans, however, even the "safety" factor notion can hardly be employed. A design specification of extremely heavy exceptional vehicles covering the whole bridge will lead to unreasonably heavy bridge members. All design specifications materially discount such traffic loads. The reductions are often so large as to make terms like "safety factor" and "overload capacity" quite meaningless. To carry the solution of this problem further, it seems that minute hazards of overloading the bridge and even of its collapse must be taken into account by means of an economic minimum criterion similar to that just explained.

Principles for the determination of design load for short and long bridge spans are already treated in the literature. This paper illustrates one additional aspect and deals only with the determination of design traffic loads for long spans from the point of view of making the probabilities for surpassing these load non-zero but small.

Present Average Long Lane Loads

The term "lane load" or "long lane load" will here designate the average load per linear foot of a continuous vehicle-lane of indefinite length, occurring for instance on a congested street.

For the long truck and bus lanes on the lower deck of the San Francisco - Oakland Bay Bridge the authors of a recent paper[2] show in their Table 3 that the average truck lane load maximum occurs at zero speed, and that it amounts to 472 lb per lin. ft of lane. This intensity, which excludes automobiles, is also found to be representative of convoys that normally use the highways. At a higher speed, say 30 miles per hour, the lane load decreases to a fraction, namely to 120 lb per ft, because of increased following distances between the trucks.

Thus for medium and long span bridges the zero speed load is always critical. Whenever this stationary load is applied alone or in combination with additional heavy trucks, no impact needs to be considered, since all vehicles are at rest.

On general highways automobiles outnumber trucks in the ratio 4:1. The average lane load at zero speed will be from 150 to 250 lb per ft of lane including trucks.[2] The former figure, 150, will presumably not be exceeded for an automobile lane without trucks and buses.

Future Long Lane Loads

To meet unknown future conditions all automobiles could be excluded. This condition would, however, be rather unlikely, except of course for lanes open to trucks only. For an alternative view the author would first look into the meaning of "future". Clearly the word means nothing beyond the lifetime of the bridge. Considering further that one dollar will grow to 11.5 dollars after 50 years of 5% compound interest, it seems futile now to build for excessive and perhaps uncertain purposes which in any case will not become material until after 50 years. It seems wiser to use the money for other needs which should be filled right now and to rebuild or strengthen the bridge after 50 years, or when needed. "Future" in this context thus would generally mean

2. "Live Loading for Long-Span Highway Bridges", by R. J. Ivy, T. Y. Lin, Stewart Mitchell, N. C. Raab, V. J. Richey, and C. F. Scheffey, Proceedings ASCE, Vol. 79, Separate No. 198, June 1953.

the next fifty years or perhaps an even shorter part of the life of the bridge.

The author thinks that a future increase in the relative proportion of trucks from 1:4 to 1:2 is ample. In conjunction with this, the average truck load 472 lb per ft on the San Francisco - Oakland Bay Bridge mentioned above could however be increased to, say, 600 lb per ft of lane, to provide for increased future use of trucks approaching the official upper weight limits, and for a modest possible increase in these limits. This would fix the future average general highway load of long stationary lanes to $(600 + 2 \cdot 150)/3 = 300$ lb per ft. Assuming an average vehicle spacing of $s = 33.3$ ft, the weight of the average usual vehicle will be $U = 10$ kips. A substantial increase in this load intensity would imply the strengthening of the main structures of most existing long-span highway bridges.

Choice of Sporadic Heavy Vehicles

However, a future increase in the design weight of sporadic exceptionally heavy vehicles will not be overlooked here. In addition to a distributed load in most unfavorable position on the free areas of the bridge floor, present German bridges of the heaviest type must be designed for one heavy truck covering 10 by 20 ft and weighing 132 kips. Some British bridge projects have been designed for an 180-ton truck, occupying a larger area and crossing the bridge once or twice during its lifetime, with stresses slightly higher than the yield-point.

Such heavy single design vehicles seem unwarranted: This discussion will consider as a largest design vehicle an extraordinary or a special-permit transport weighing $T = 130$ kips and occupying the same lane length of $s = 33.3$ ft including following distances, or 3900 lb per ft of lane, Fig. 1. When such a transport appears in lanes with the zero speed loads just given, no impact will be added. To a single transport (say, three 43 kip axles at 6 ft distances) moving without "usual" vehicles U, Fig. 2, an impact of 20% may be added, and to a single axle of 45 kips, Fig. 3, an impact of 40%.

Where $T = 130$ kips is considered excessive, an analysis analogous to the following can easily be carried through for any other weight T. Any specific design vehicle load for short spans can be logically fitted to any reasonable average long lane load by a proper choice of the heavy transport load T. It is only necessary to choose T so high that for one span-length the stresses due to the U, T-combination are equal to the highest stresses due to the (one, two, or three) specified design vehicles and higher for all other span-lengths. The specification vehicles may have concentrated wheel and axle loads. All bridge spans or bridge members smaller than this span-length will here be termed "short". Short members should be designed for the specified design vehicles rather than for the fitted U, T-combination.

If the specification design load is not simply one vehicle T, such fitting is always possible according to the following method. A proper transition can be determined between the design load for short spans and the ultimate long lane traffic load U known from observations on heavy truck and bus lanes. Such fitting may thus be required in the reduction of long-span live loads implied in some modern bridge specifications, e.g. those of AASHO. Here it is assumed that the heavy transport weight $T = 130$ kips is the outcome of a proper fitting to some specified shortspan design vehicles or of some other rational considerations.

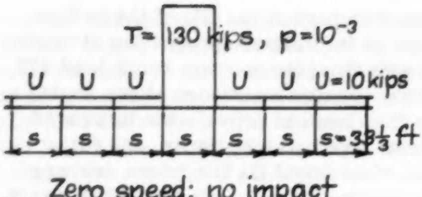

Fig. 1. U, T-load

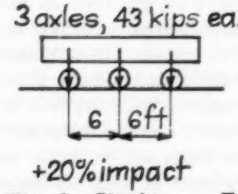

Fig. 2. Single transport

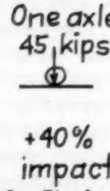

Fig. 3. Single axle

Frequency of Heavy Vehicles

By specifying one vehicle in one thousand to be such a sporadic heavy transport, a "future" increase in the frequency of special-permit transports is presumably well taken care of, since such permits now probably do not exceed one for every ten thousand vehicles.

Probabilities for Heavy Loads

The probability for one heavy transport T to cover a bridge span equal to one vehicle spacing, is one in one thousand cases of loadings, or $0.001 = p$. The load intensity is then $T = 130$ kips per vehicle spacing (or 3900 lb per ft, at a spacing of $s = 33.3$ ft).

The probability is $2\,p\,(1 - p) = 2 \times 0.001 \times 0.999$ for one but not two transports T to occupy either spacing on a bridge span equal to two vehicle spacings. This gives an average stationary load intensity of $(U + T)/2 = (10 + 130)/2 = 70$ kips per spacing. The probability is p^2 for two transports T to occupy two adjacent vehicle spacings and the load intensity is again 130 kips per spacing.

The probability is $P = \binom{n}{t} p^t (1 - p)^{n-t} = \binom{n}{t} 0.001^t \times 0.999^{n-t}$ for t heavy transports T to occupy any t spaces in a lane of n adjacent spaces, the binomial coefficient $n!/t!\,(t - n)!$ being denoted $\binom{n}{t}$. This yields an average load intensity of $A = U + (T - U)\,t/n = (10 + 120\,t/n)$ kips per vehicle spacing, to which no impact shall be added, since the lane is stationary. Moving lanes demand considerably larger spacings s. If n is quite small, the effect of a single transport T moving with impact should also be investigated.

The probabilities P for a definite number of heavy transports $t = 0, 1, 2, 3, 4$ to fall within n adjacent vehicle spacings are plotted in a diagram, Fig. 4. At every point n, t the corresponding average load A can be determined or, more generally only t/n of the formula $A = U + (T - U)\,t/n$.

Fig. 4 then discloses, for instance, that an average load of 19 kips per spacing is not exceeded more than once in one hundred thousand cases of stationary continuous loadings ("traffic jams") on a bridge span 40 vehicle-spacings long (since $P = 10^{-5}$ is the probability for three heavy transports to amass in 40 spacings, giving an average load intensity of $A = 10 + 120 \times 3/40 = 19$). To be sure, other loadings with 5 up to 40 heavy transports add to this probability, but very little: the probabilities to be added form a roughly geometric series with a ratio of about 1/1,000, making the total probability
$P < \binom{n}{t} p^t (1 - p)^{n-t-1.5}$.

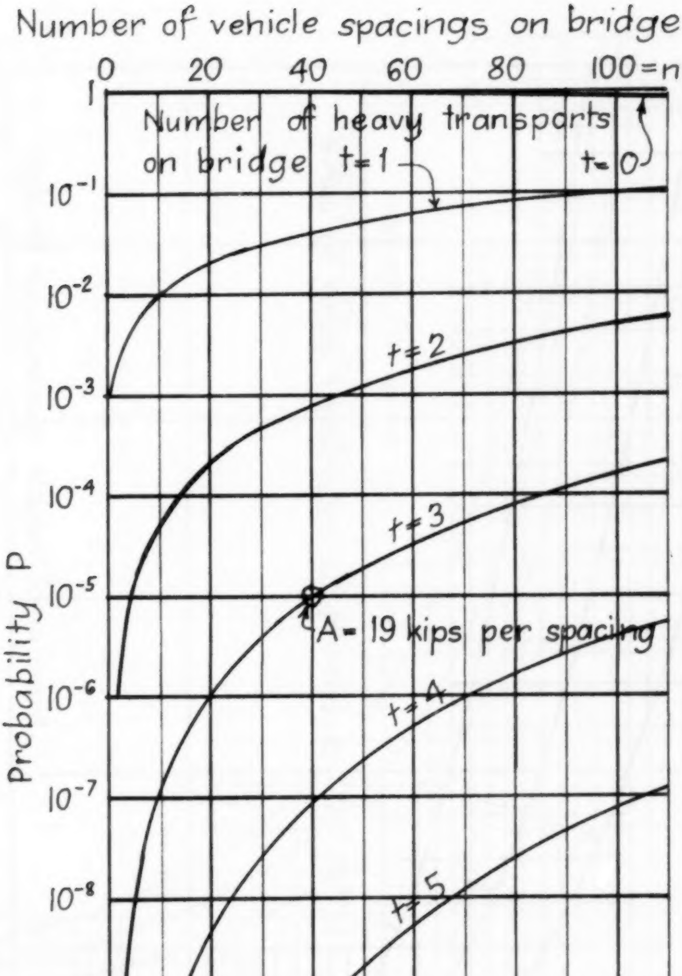

Fig. 4. Probabilities for coincidence of heavy transports.

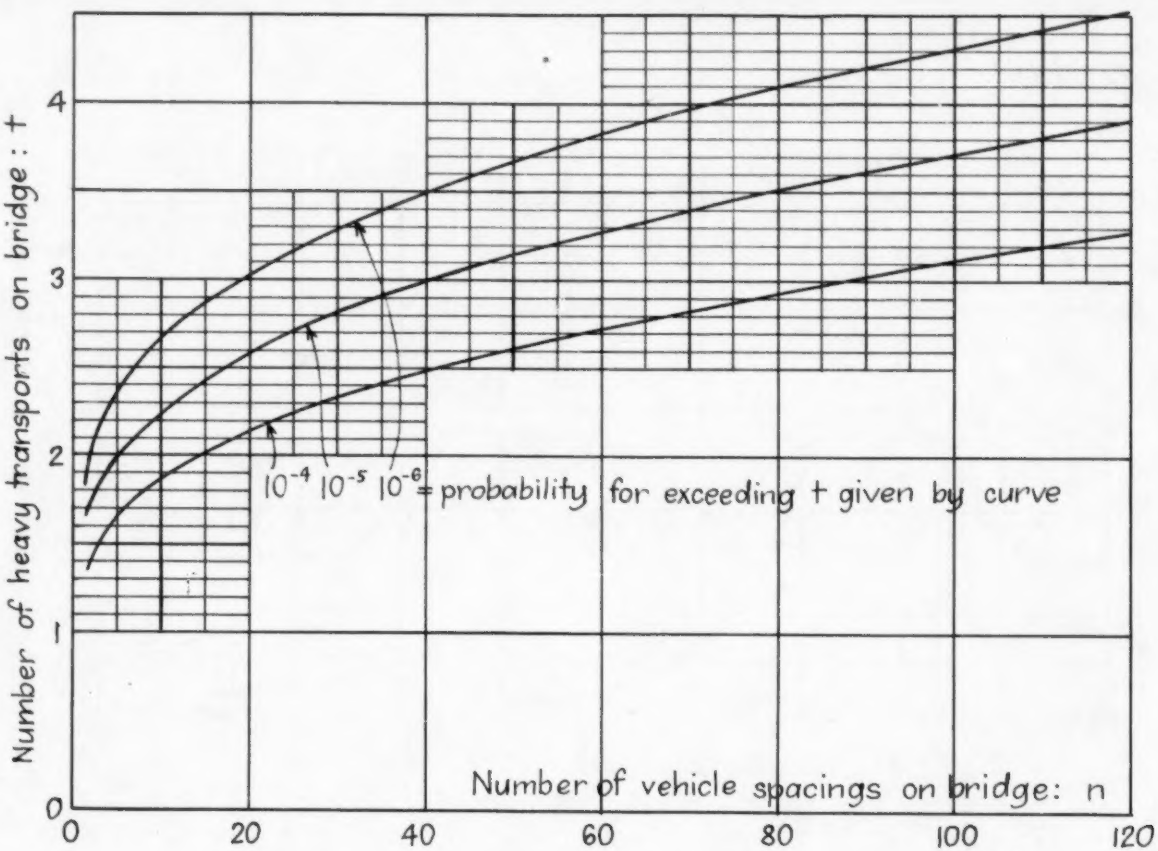

Fig. 5. Probabilities for exceeding t.

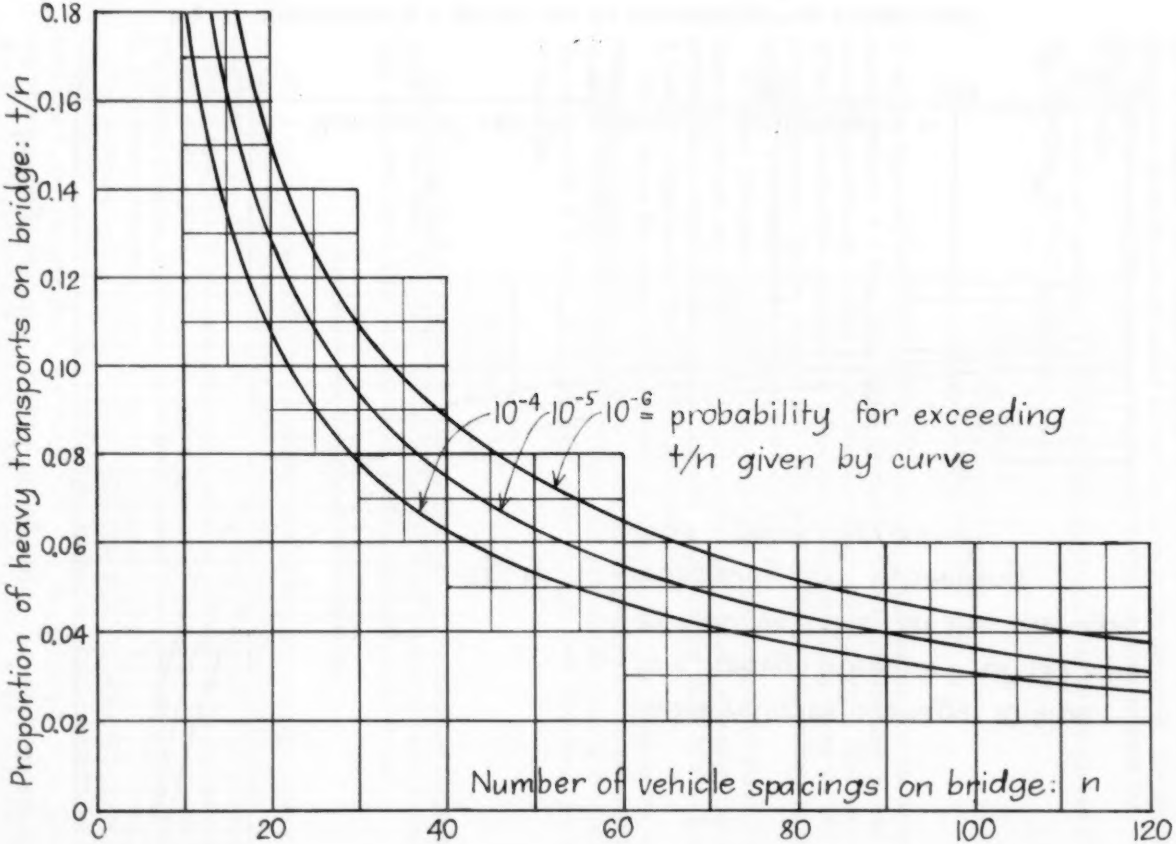

Fig. 6. Probabilities for exceeding t/n.

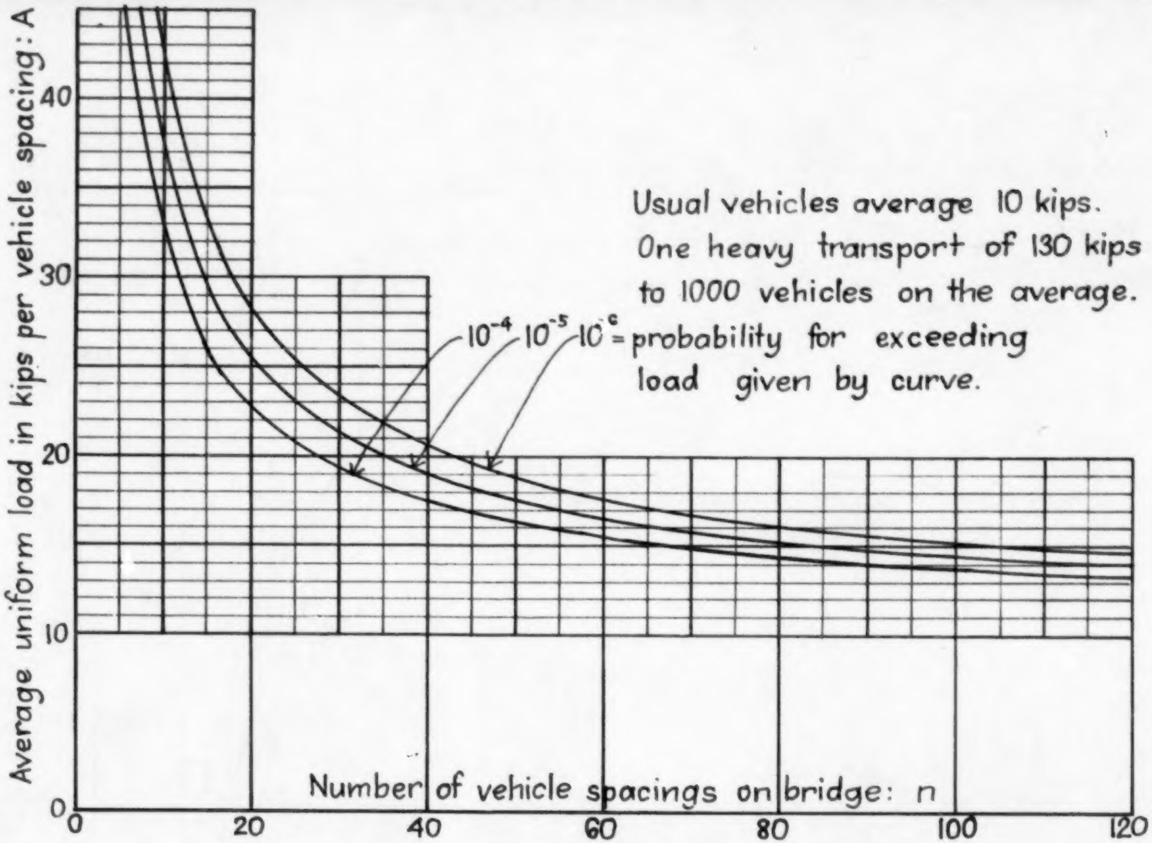

Fig. 7. Probabilities in a special case for exceeding various average loads.

To avoid discontinuous changes of probabilities it is appropriate and on the safe side to interpolate between the whole numbers t in Fig. 4, thus admitting fractions of vehicles. The diagram can then be redrawn to show, as in Fig. 5, for each number n of vehicle spacings on the bridge, the number t of heavy transports which is not exceeded more than once in ten thousand, one hundred thousand, or one million cases of stationary traffic congestions upon the bridge span.

For values of t between 2 and 4.5 a close approximation to the curves of Fig. 5 is ($\log_{10} t! = -0.8 + 0.55t$)

$$t = \frac{-\log_{10} P + 0.8}{-\log_{10} p - \log_{10} n + 0.55}$$

It will be apparent from the concluding remarks of this paper that the determining factors: n = total lane length divided by the vehicle spacing, p, and P, cannot be judiciously chosen except within rather wide margins. The accuracy of the approximation just made is therefore more than satisfactory for the present purpose. On the other hand traffic load specifications seldom need revision: a formula for t will be used very rarely.

At these infrequent occasions it is preferable from the point of view of a thorough insight into the problem at each revision to treat the problem by the original more exact procedure of constructing curves like those in Fig. 5 and 6, without reliance upon the approximate formula.

Fig. 6 shows the proportion t/n of heavy transports. Fig. 7 shows the corresponding average load $A = U + (T - U)\, t/n$ in the special case treated before, namely U = 10 kips, T = 130 kips, $p = 10^{-3}$, and one vehicle spacing s = 33.3 ft. The curves for $P = 10^{-4}$, 10^{-5}, and 10^{-6} differ by less than what could perhaps be expected.

Traffic on Several Lanes

The average load on two or more adjacent lanes can be obtained from the curve for one lane, simply by counting n as the total number of vehicle spacings contained in all the lanes covering the loaded length in the bridge.

Increment for Equivalent Moment Load

For a more correct value of equivalent loading than the average loading A, an increment for equivalent load over average load can be derived, Fig. 8: The weight $(T - A)/(n - 1)$ is removed from the average weight A in every one of the n_1 spacings in one lane and $n_1(T - A)/(n_1 - 1)$ added to the remaining weight $A - (T - A)/(n_1 - 1)$ in the center spacing of the bridge span, Fig. 8a. The center spacing will then carry a total load of $A + (n_1 - 1)(T - A)/(n_1 - 1) = T$, or a heavy transport causing maximum moment, Fig. 8c.

The increase $n_1(T - A)/(n_1 - 1)$ will cause the same moment $M = (1/2n_1 - 1/4)s \cdot 1/2n_1(T - A)/(n_1 - 1)$ in a simple span as do n uniformly distributed vehicles V, or $1/4n_1 s \cdot 1/2n_1 V$, so that $V = (T - A)\,(2n_1 - 1)/n_1(n_1 - 1)$. The equivalent uniform loading for moment will be $E_M = A - (T - A)/(n_1 - 1) + V = A + (T - A)/n_1$ per vehicle spacing. The average loading A thus should be increased by the simple expression $(T - A)/n_1$ for equivalent loading for moment, Fig. 8c.

Increment for Equivalent Shear Load

For maximum shear, Fig. 9, the removed load of $(T-A)n_1/(n_1-1)$ is instead added to the remaining weight $T - (T - A)/(n_1 - 1)$ in the vehicle spacing

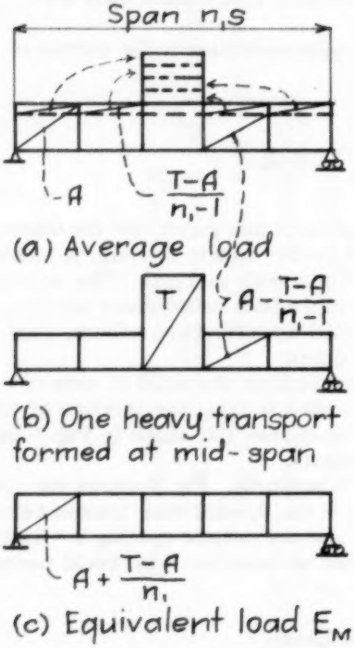

Fig. 8. Equivalent load for moment

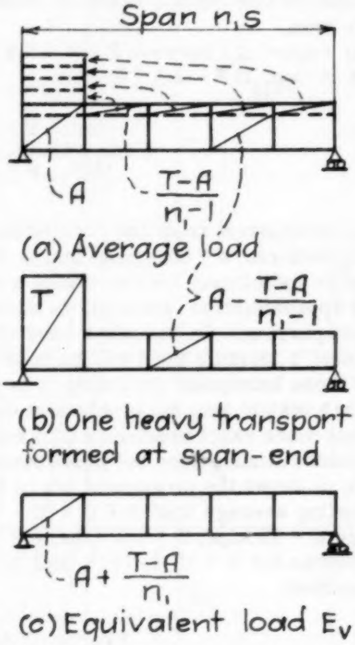

Fig. 9. Equivalent load for shear

nearest to the end of the span, Fig. 9a, making its total load equal to a heavy transport T, Fig. 9b. This addition produces a shear of $(T - A)[n_1/(n_1 - 1)]/[(n_1-1/2)/n_1]$ which is equal to that, $1/2n_1V$, caused by n uniformly spaced vehicles of weight V. Thus $V = (T - A)(2n_1 - 1)/n_1(n_1 - 1)$ as before, so that the equivalent uniform loading for shear $E_V = A + (T - A)/n_1$ per vehicle spacing, Fig. 9c, will be exactly the same as for moment.

The equivalent uniform load E for maximum moment or shear should be understood as being more improbable or infrequent than the average uniform loads A previously found. When a bridge is loaded with more than one lane it should suffice to add $(T - A)/n_1$ to A in the most critical lane only, n_1 being the number of vehicle spacings in that one lane of the bridge span.

Illustrative Examples

For a concrete example, consider a three-lane bridge, Fig. 10, with a span-length of 30 vehicle spacings (that is, for a spacing of s = 33.3 ft, a span length of 1,000 ft). The average uniform load which cannot be exceeded more than once in 10^5 traffic congestions at standstill is found from Fig. 7, for $n = 3 \cdot 30 = 90$, to be A = 15 kips per vehicle spacing. The equivalent uniform load E for

moment or shear in the span is obtained by adding to the uniform average load A in the most critical lane only (nearest to the main girder) a uniform load of $(T - A)/n_1 = (130 - 15)/30 = 3.84$ kips per spacing. The main girder thus should be designed for uniform lane loads of 15, 15, and, nearest to the main

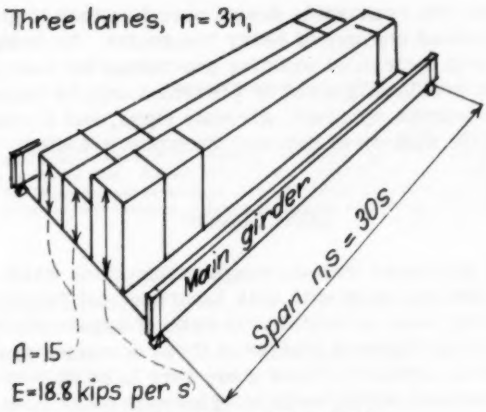

Fig. 10. Main girder load

girder, 18.84 kips per vehicle spacing (for $s = 33.3$ ft: 450, 450, and 565 lb per lin. ft of lane). No impact should be added.

A decrease to 10^{-6} in the probability of overloading would increase A by less than 6% to 15.8 kips per spacing, and the equivalent load, for the most critical lane, to $E = 15.8 + 3.82 \quad 19.6$ kips per spacing.

A three-lane bridge with a span of $n_1 = 9$ spacings (say 300 ft) similarly yields for $P = 10^{-5} (n = 27)$ the result $A = 20.6$, and, in the most critical lane only, $E = 20.6 + (130 - 20.6)/9 = 32.8$ kips per spacing (or, for $s = 33.3$ ft, $A = 618$, $E = 955$ lb per lin. ft. of lane).

Because of the heavy special-permit loadings assumed, these uniform loads for a 300 ft span are higher than those specified by AASHA, while the loads deduced above for a 1000 ft span are much smaller.

Short spans of say less than 3s or 100 ft length should preferably be designed directly with the axle loads and spacing of design vehicles corresponding to the heaviest actual vehicles that are expected to load the bridges.

Choice of Probability Limits

Which probability is to be used in the determination of bridge loads depends primarily on the expected frequency of stationary traffic congestions including at least one heavy transport T in critical position for moment or shear. In a period of say 30 years a city bridge with continuous traffic can be expected to sustain perhaps 10^6 stationary traffic congestions, while a rural bridge may be jammed 10^4 times or less.

Ten stationary congestions every day make 10^5 in 27 years, including one probable case of design loading $A + T (T - A)/n$ for $P = 10^{-5}$, or an even smaller probability because of the condition that one of the heavy transports shall be in critical position for moment or shear. On the average of once in every 270 years the traffic load on a two lane 300 ft span will be increased by more than 12% above the former load. This would appear to the writer to be

a conservative starting-point for the determination of traffic loads for long-span bridges.

Organized Convoys of Heavy Transports

It is impossible with reasonable design specifications to guard against damage from organized convoys of heavy transports. No bridge specifications do that, even in their most exacting provisions for heavy long lane loads. Systematic overloadings can be prevented only by regulations of maximum loads and minimum spacings, adequate signs, and a confidence in the common sense of the drivers of colossal transport vehicles.

CONCLUSION

The results of this paper with its rough assumptions which are presumably on the safe side, are in accord with both American and Swiss reductions of traffic loads for long lanes of loading. In spite of apparently low traffic loads on long lanes, the long-spanned bridges of these countries are seen by probability considerations similar to those made here to be able to carry very heavy transport vehicles statistically mingled with other road traffic, provided only that the immediate load supporting elements, such as roadway slab, secondary girders, cross girders, and hangers, are strong enough.

PROCEEDINGS-SEPARATES

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c. Discussion of several papers, grouped by Divisions.
d. Presented at the Atlanta (Ga.) Convention of the Society in February, 1954.
e. Presented at the Atlantic City (N.J.) Convention in June, 1954.